§7.1 EQUIVALENT FORMS OF EXPONENTIAL EQUATIONS

1. Write each expression as a power of the base indicated.

a) 64^3, base 4 **b)** $\left(\frac{1}{4}\right)^5$, base 2 **c)** 5^3, base 4

2. Write each expression as a single power of 3.

a) $\sqrt[3]{81}$ **b)** $\frac{\sqrt{27}}{\sqrt[4]{9}}$

3. Solve.

a) $7^x = 49^{x+5}$ **b)** $4^{t-3} = 32^{\frac{t+2}{3}}$ **c)** $36^{3x-1} = 216^{5-x}$

4. Consider the equation $5^{3x-1} = 125^{2x}$.

a) Solve this equation by expressing both sides as powers of a common base.

b) Solve the same equation by taking the logarithm, base 5, of each side.

5. Solve $\left(\sqrt{125}\right)^{x-3} = 25^{2-x}$.

6. a) Solve. Give exact answers.

i) $5 = 10^x$ **ii)** $3 = 10^x$ **iii)** $7 = 10^x$

b) Use your answers to part a) to state a formula that could be used to solve $b = 10^x$ for *x*.

7. a) Solve $16^{3x+2} = 64^{5-3x}$ by expressing both sides of the equation as powers of 4.

b) Solve $16^{3x+2} = 64^{5-3x}$ by expressing both sides of the equation as powers of 2.

§7.2 TECHNIQUES FOR SOLVING EXPONENTIAL EQUATIONS

1. Solve for *t*. Round answers to two decimal places.

a) $5 = (1.12)^t$

b) $7 = 2(1.06)^{2t+1}$

c) $4 = \left(\frac{1}{8}\right)^{t-3}$

d) $6.5 = 1.5\left(\frac{1}{2}\right)^{\frac{t}{15}}$

2. The amplitude of a vibration decays according to the equation $a = 4.5(0.65)^{\frac{t}{60}}$, where *a* is the size of the amplitude, in centimetres, at time *t*, in seconds.

a) What will the amplitude be after 30 s, to the nearest tenth of a centimetre?

b) How long (to the nearest second) will it take the amplitude to decay to 30% of its original value?

c) Would it take twice as long as your answer for part b) for the amplitude to decay to 15% of its original value? Explain.

3. Solve each equation. Leave answers in exact form.

a) $3^{x-3} = 4^{x-4}$

b) $7^{2x-3} = 2^{-3x}$

4. Consider the equation $3^{2x} - 4(3)^x + 1 = 0$.

a) Write the equation in the form $az^2 + bz + c = 0$, where $z = 3^x$.

b) Solve the equation using the quadratic formula.

c) Identify any extraneous roots.

The model for **Exponential Growth** or **Decay** is ***$A(t) = A_0(r)^{t/p}$***

where, $A(t)$ = amount as a function of time (final amount)
A_0 = intial amount (@ time 0)
r = the ratio/multiplier of growth or decay
t = time (must be the same unit of time as the period)
p = period of time for the growth or decay to occur

The **half-life** of a radioactive substance is the time required for a sample of the material to decay to half of its initial amount.

5. A 25-mg sample of radioactive francium decays to 20 mg in 13.6 min. Determine the half-life of francium.

6. Solve and check for extraneous roots. Leave answers in exact form.

a) $3^{2x} - 6(3)^x - 27 = 0$

b) $2^x = 2 + 3(2)^{-x}$

c) $5^{2x} + 1 = 3(5)^x$

7. The population of a colony of bacteria grows according to the formula $P(t) = 4(1.40)^{\frac{t}{24}}$, where *P* is the population, in thousands, and *t* is the time, in hours.

a) How long does it take the population to reach 10 000, to the nearest hour?

b) Calculate the time it takes for the population to double, to the nearest hour.

8. The maximum height that a ball reaches after bounce number *n* is given by the equation $H = 2.0(0.90)^n$, where *H* is the height, in metres.

a) What is the ball's maximum height after the fifth bounce?

b) What is the first bounce after which the maximum height is less than 10 cm?

9. Solve $4^x + 2^{x+2} = 32$.

10. Rewrite the equation $P(t)=4(1.40)^{\frac{t}{24}}$ with base 1.40 replaced with 2.

11. The general equation for population growth is $P(t)=P_0\left(1+\frac{R}{100}\right)^{\frac{t}{t_0}}$, where *R* is the growth rate, in percent, over time period t_0. Suppose a population grew from 10 000 to 25 000 in six years. If time is measured in years, calculate

a) the yearly growth rate

b) the growth rate per decade (10 years)

§7.3 PRODUCT AND QUOTIENT LAWS OF LOGARITHMS

KEY CONCEPTS

- The **product law of logarithms** states that $\log_b x + \log_b y = \log_b(xy)$ for b > 0, b ≠ 1, x > 0, y > 0.
- The quotient law of logarithms states that $\log_b x - \log_b y = \log_b(\frac{x}{y})$ for b > 0, b ≠ 1, x > 0, y > 0.
- The laws of logarithms can be used to simplify expressions and solve equations.

Examples:

1. Simplify, using the laws of logarithms. Then evaluate, correct to three decimal places.

a) $\log 99 - \log 9$

b) $\log_5 20 + \log_5 3$

2. Simplify each algebraic expression. State any restrictions on the variables.

a) $\log_4 x + \log_4(2y) - \log_4(z-3)$

b) $\log a - 2\log b + 3\log c$

3. Evaluate, using the laws of logarithms.

a) $\log_3 54 - \log_3 6$

b) $\log_6 4 + \log_6 6 + \log_6 \frac{3}{2}$

c) $\log 5 + 2\log 4 - \log 8$

4. Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log\left(\frac{x^2 y}{z^3}\right)$

b) $\log_3\left(\frac{5\sqrt[3]{m}}{n^4}\right)$

5. Simplify. State any restrictions on the variables.

a) $\log(m^5) - \log(m^2) + \log m$

b) $\log\left(\sqrt[3]{p}\right) + \log\left(\sqrt{p}\right) + \log\left(\sqrt[6]{p}\right)$

c) $\log(x^2 - 5x - 6) - \log(x - 3)$

d) $\log(6x^2 + 5x - 6) + \log(2x - 3) - \log(4x^2 - 9)$

6. Use the laws of logarithms to write *y* as a function of *x* for each of the following. Then, state the domain of the function.

a) $\log(xy) = 2\log(x - 3)$

b) $\log(y) + 3 = \log(y + 1) + \log(x)$

c) $\log\left(\frac{x^2}{y}\right) = 2\log(x + 5)$

7. Prove that $\dfrac{1}{\log_x 10}+\dfrac{1}{\log_y 10}=\dfrac{1}{\log_{xy} 10}$. (Hint: recall that $\log_n m=\dfrac{\log m}{\log n}$)

§7.4 TECHNIQUES FOR SOLVING LOGARITHMIC EQUATIONS

1. Solve algebraically.

a) $\log(x+5)=2$

b) $1-2\log(p-8)=0$

c) $\log_2(m+12)=4$

2. Use Technology Solve using graphing technology.

a) $\log(x-2)=1$

b) $2-\log(v+1)=0$

3. Solve. Identify and reject any extraneous roots.

a) $\log_2(x-5)+\log_2(x-2)=2$

b) $\log x+3=\log(10x^3)$

c) $2\log m + 3\log m = 10$

4. Solve. Check for extraneous roots.

a) $\log(x+5)^3 = 3$

b) $\log\sqrt{x^2 - 5x + 9} = 1$

5. Solve.

a) $\log_2 x + \log_2(x-3) = 2$

b) $\log(x+2) = 2 - \log(x-3)$

6. The number of years, *n*, required for an investment P_1 to grow to amount P_2 when interest is *i*% per year (expressed as a decimal) is given by the formula $n = \dfrac{\log P_2 - \log P_1}{\log(1+i)}$.

a) How many years does it take for $1500 to grow to $2380 at a yearly interest rate of 8%?

b) How much should be invested, to the nearest dollar, in order for it to grow to be $4137 in 8 years at a yearly rate of 5%?

c) What yearly interest rate is needed to allow $900 to grow to $1689 in 10 years, to the nearest tenth of a percent?

7. Solve $\log_2 \sqrt{x} + \log_4 x = 5$.

9. **a)** Calculate $(\log_2 5)(\log_5 8)(\log_8 10)$.

b) Compare your answer from part a) to $\log_2 10$.

c) Prove that $(\log_a b)(\log_b c)(\log_c d) = \log_a d$